Helburuetara orientatutako egokitzapena denboran aurrera doazen problema pseudo-dualak erabilita

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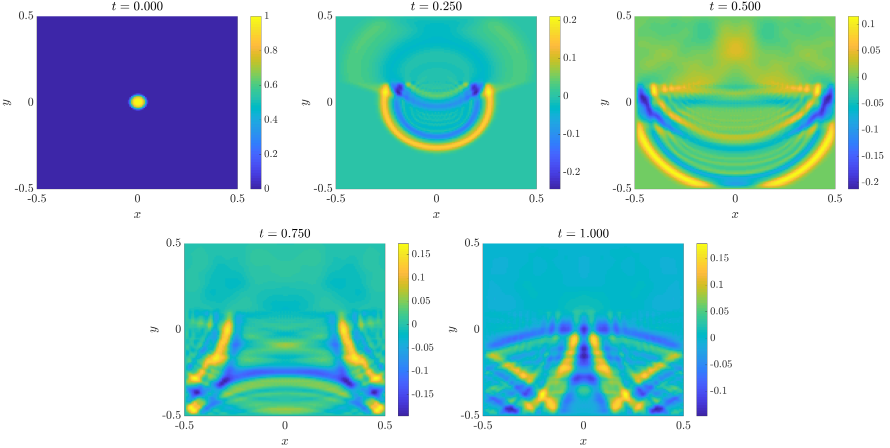
2020ko uztailak 10
Matematikari Euskaldunen IV. Topaketa
Eibar

Deribatu Partzialeko Ekuazioak (DPE)

Uhin-ekuazioa: $u_{tt} - \nabla \cdot (\alpha \nabla u) = f$

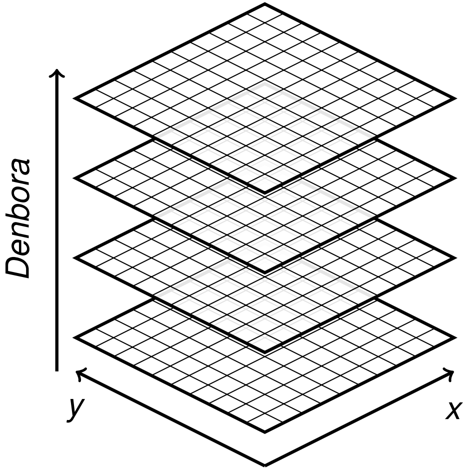


Diskretizazioa denboraren eremuko problemetan

Lerroen Metodoa:

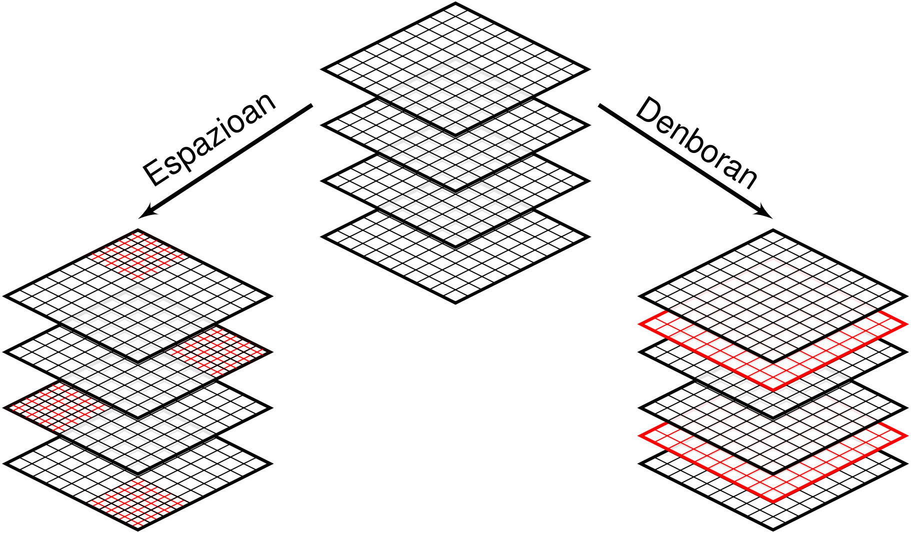
- **EFM** espazioan
- **EDA** sistema denboran

$$(M + \tau K)u^k = Mu^{k-1} + F$$



Sarearen egokitzapena

Helburua: Errore globala murriztea

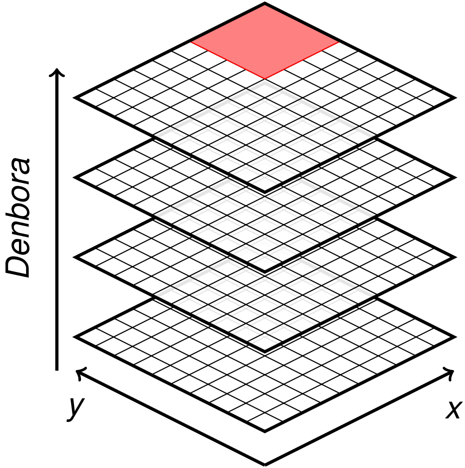


Helburuetara orientatutako egokitzapena

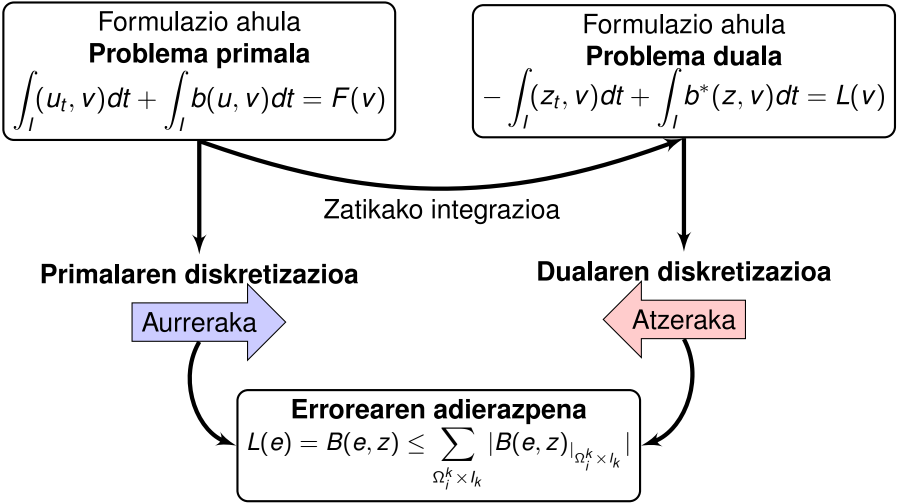
Errorea: $e = u - u_h$

Interes-kantitatea:

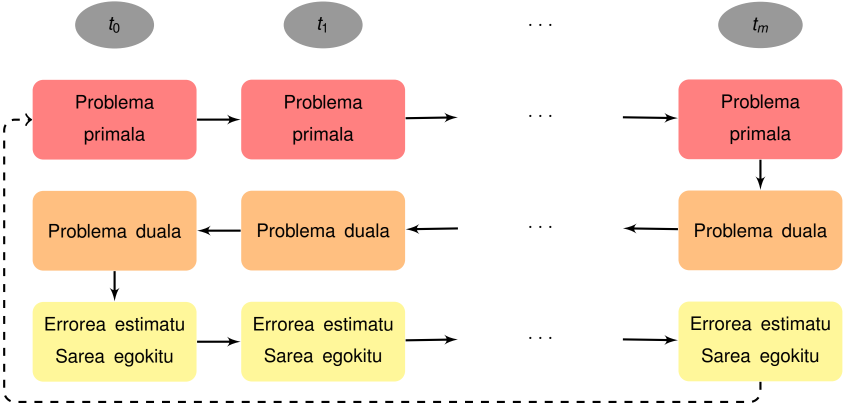
$$L(e) = \int_{\Omega_0} e(x, y, T)\, d\Omega$$



Zein osagai behar ditugu?



Algoritmo klasikoa



Helburu nagusia

Algoritmo klasikoaren **arazo nagusia**:

Problema primala eta dualaren soluzioak gordetzea iterazio bakoitzean

Helburu nagusia

Helburuetara orientatutako egokitzapena AURRERANTZ egitea

J. Muñoz-Matute, D. Pardo, V. M. Calo and E. Alberdi,
Forward-in-time goal oriented adaptivity,
International Journal for Numerical Methods in Engineering, 2019, vol. 119, p. 490-505.

Problema pseudo-duala

Problema pseudo-duala

Bilatu $\tilde{z} \in \mathcal{U}$ non

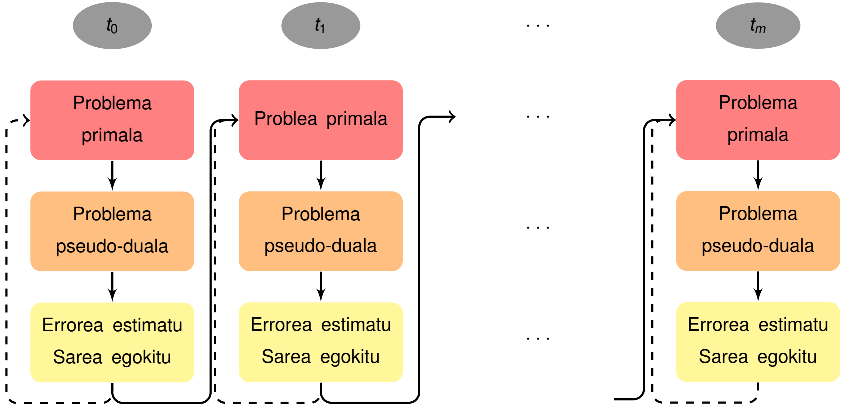
$$\tilde{B}(\tilde{z}, v) = L(v), \ \forall v \in \mathcal{V}$$

Errorearen adierazpen berria:

$$|L(e)| = |\tilde{B}(\tilde{z}, e)| \leq \sum_{\Omega_i^k \times I_k} |\tilde{B}(\tilde{z}, e)_{\Omega_i^k \times I_k}|$$

V. Darrigrand, D. Pardo and I. Muga,
Goal-oriented adaptivity using unconventional error representations for the 1D Helmholtz equation,
Computers & Mathematics with Applications 69(9), 964-979 (2015)

Algoritmo berria



Beroaren ekuazioa

Ekuazioa: $u_t - \nabla \cdot (\kappa \nabla u) = f$, $\Omega = [0, 1]$, $T = 1$

Iturria: $f(x, t) = (1 + \pi^2 t) \sin(\pi x)$

Hasierako baldintza: $u(0) = 0$

Difusio koefizientea:

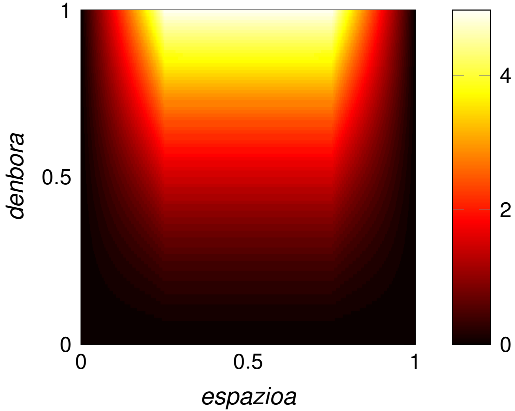
$$\kappa(x) = \begin{cases} 10, & x \in [0.25, 0.75] \\ 0.01, & \text{bestela} \end{cases}$$

Interes kantitatea:

$$L(u) = \int_I \int_{\Omega_0} u(x, t)\, dxdt$$

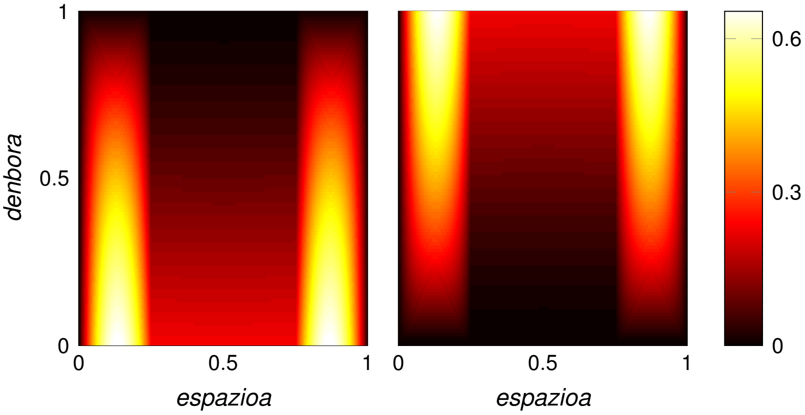
non $\Omega_0 = (0, 0.25) \cup (0.75, 1) \subset \Omega$

Problema primala



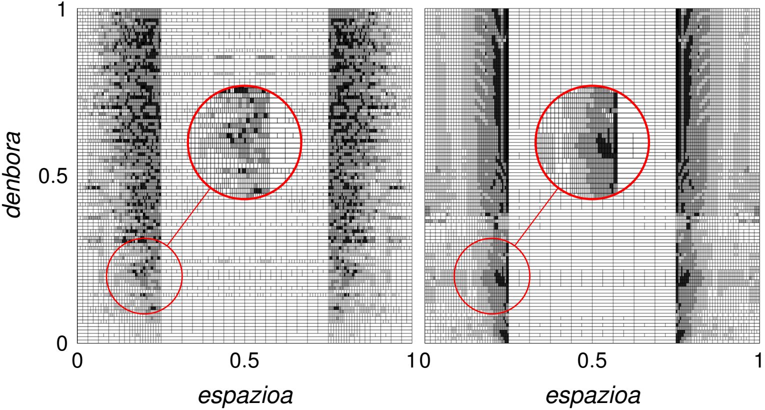
Problema primala

Problema duala eta pseudo-duala



(a) Problema duala (b) Problema pseudo-duala

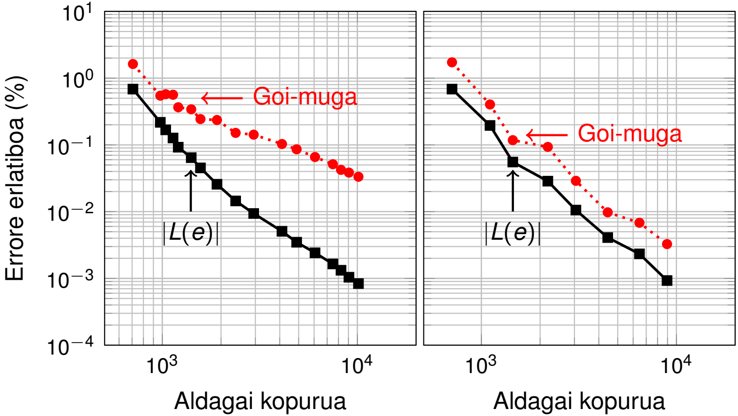
Egokitatutako sareak



(a) Algoritmo klasikoa

(b) Algoritmo berria

Errorearen estimazioa



(a) Algoritmo klasikoa (b) Algoritmo berria

Adbekzio-difusio ekuazioa

Ekuazioa: $u_t - \nabla \cdot (\kappa \nabla u) + a \cdot \nabla u = f$, $\Omega = [0,1]$, $T = 0.25$

Iturria: $f(x,t) = 0$

Adbekzio eta difusio koefizienteak: $\kappa = 0.025$, $a = 2.5$

Hasierako baldintza:

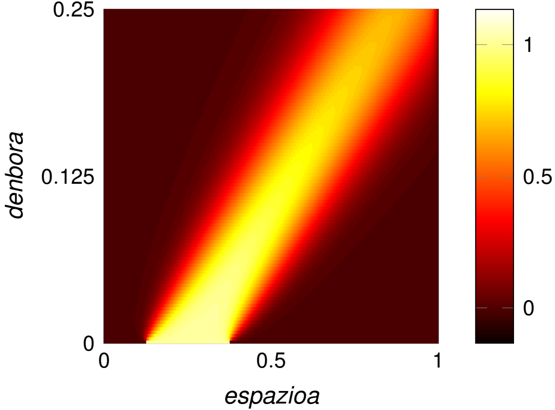
$$u_0(x) = \begin{cases} 1, & x \in [0.125, 0.375] \\ 0, & \text{bestela} \end{cases}$$

Interes kantitatea:

$$L(u) = \int_{I_0} \int_{\Omega_0} u(x,t)\, dxdt,$$

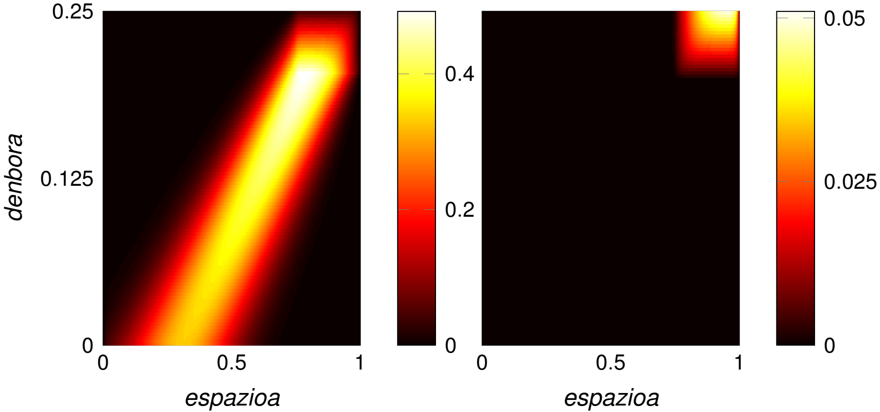
non $I_0 \times \Omega_0 = (0.75, 1) \times (0.2, 0.25) \subset I \times \Omega$.

Problema primala



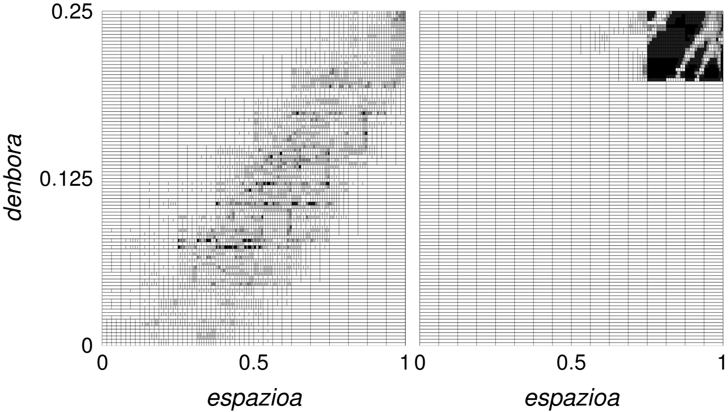
Problema primala

Problema duala eta pseudo-duala



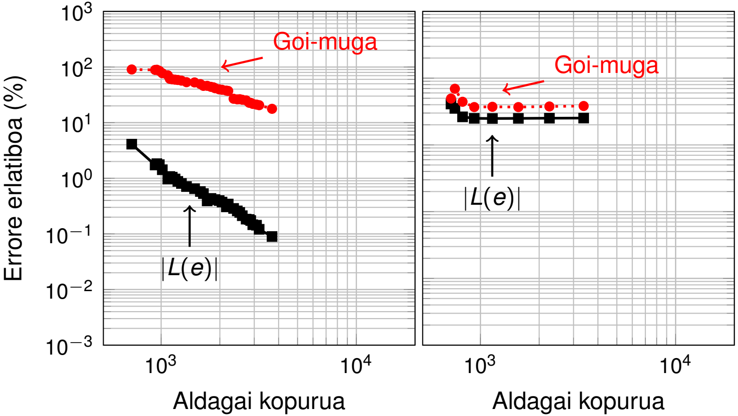
(a) Problema duala (b) Problema pseudo-dual

Egokitatutako sareak



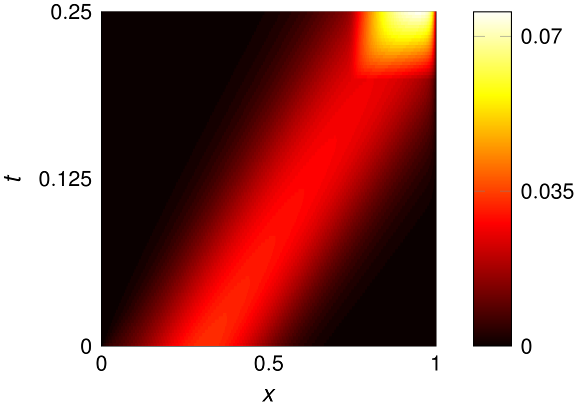
(a) Algoritmo klasikoa (b) Algoritmo berria

Errorearen adierazpena



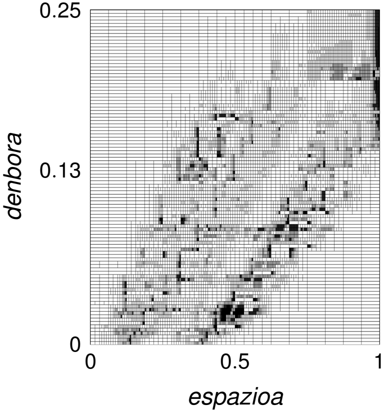
(a) Algoritmo klasikoa (b) Algoritmo berria

Problema pseudo-duala

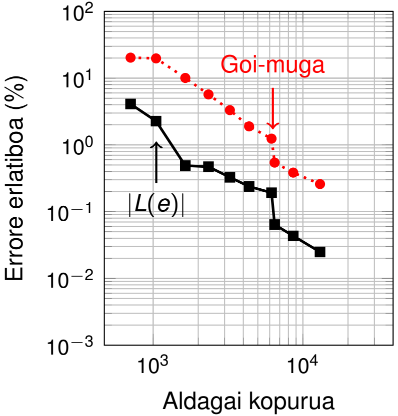


Problema pseudo-duala

Egokitatutako sarea eta errorearen adierazpena



(a) Egokitatutako sarea



(b) Errorearen adierazpena

ESKERRIK ASKO!